Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ if A and B are dependent

$P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are disjoint events,

$P(A \text{ and } B) = 0$

Math 119 **Review Set 3** **Name:** Key

1. A candy dish contains three red candies, 4 yellow candies and 5 blue candies. You close your eyes, choose two candies <u>one at a time</u> (without replacement) from the dish, and record their colors.

   (a) Find the probability that both candies are red. (a) $\boxed{0.0455}$

Let A = 1st candy is red. Let B = 2nd candy is red. Then,

$$P(\text{both red}) = P(A \text{ and } B)$$
$$= P(A) \cdot P(B|A) = \frac{3}{12} \cdot \frac{2}{11} = \frac{6}{132}$$

   (b) Find the probability that both candies are not blue. (b) $\boxed{0.3182}$

Let A = 1st candy is not red. Let B = 2nd candy not red.

Then, $P(\text{both not blue}) = P(A \text{ and } B)$
$$= P(A) \cdot P(B|A) = \frac{7}{12} \cdot \frac{6}{11} = \frac{42}{132}$$

2. The proportions of blood phenotypes A, B, AB and O in the population of all Caucasions in the U.S. are reported at 0.41, 0.10, 0.04 and 0.45, respectively. An individual is randomly selected from the population. Find the probability that the person will have either type A or type AB blood.

2. $\boxed{0.4500}$

0, disjoint events

$$P(A \text{ or } AB) = P(A) + P(AB) - \cancel{P(A \text{ and } AB)}$$
$$= 0.41 + 0.04 = 0.45$$

Disjoint Events have no outcomes in common. If events A and B are disjoint, then if one of the events occurs, it is impossible for the other to occur

3. Toss three fair coins. Find the probability that all three are tails.

$$P(\text{All tails}) = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8} = \boxed{0.1250}$$

each event is independent

4. In a genetics experiment, the researcher mated two *Drosophila* fruit flies and observed the traits of 300 offspring. The results are show in the table.

	Wing Size	
Eye Color	Normal	Miniature
Normal	140	6
Vermillion	3	151

One of these offspring is randomly selected and observed for the two genetic traits.

(a) Find the probability that the fly has normal eye color and normal wing size.

(a) $\boxed{0.4667}$

$$\frac{140}{300}$$

(b) Find the probability that the fly has vermillion eyes.

(b) $\boxed{0.5133}$

$$\frac{3+151}{300}$$

(c) Find the probability that the fly has either vermillion eyes or miniature wings.

$P(V \text{ or } M) = P(V) + P(M) - P(V \text{ and } M)$

(c) $\boxed{0.5333}$

$$= \frac{154}{300} + \frac{157}{300} - \frac{151}{300} = \frac{160}{300}$$

(d) Find the probability that the fly has miniature wings given that it's eye color is normal.

Let $A =$ eye color is normal

(d) $\boxed{0.0411}$

Let $B =$ mini wings

Then $$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{6}{300}}{\frac{146}{300}} = \frac{6}{\cancel{300}} \cdot \frac{\cancel{300}}{146}$$

$$= \frac{6}{146}$$

5. Two cards are drawn from a standard deck of cards, one at a time (with replacement). Find the probability that the draw includes an ace and a ten.

Let A = 1st card is an ace and the 2nd card is a 10.

Let B = 1st card is a 10 and the 2nd card is an Ace.

Then $P(A \text{ or } B) = P(A) + P(B) - \cancel{P(A \text{ and } B)} \to 0$, disjoint events

$$= P(A) + P(B) + 0$$

$$= \frac{4}{52} \cdot \frac{4}{52} + \frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2704} + \frac{16}{2704} = \frac{32}{2704} = \boxed{0.0118}$$

6. Two cards are drawn from a standard deck of cards, one at a time (without replacement). Find the probability that the draw includes an ace and a ten.

Let A and B be defined the same as before.

$$\text{Then } P(A \text{ or } B) = P(A) + P(B) - \cancel{P(A \text{ and } B)} \to 0$$

$$= P(A) + P(B)$$

$$= \frac{4}{52} \cdot \frac{4}{51} + \frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} + \frac{16}{2652} = \frac{32}{2652} = \boxed{0.0116}$$

7. Suppose that $P(A) = 0.3$ and $P(B) = 0.5$. If events A and B are disjoint events, find these probabilities:

(a) $P(A \text{ and } B) = 0$ (a) $\boxed{0}$

Disjoint events have no outcomes in common.
If events A and B are disjoint, then if one of the events occurs, it is impossible for the other event to occur simultaneously.

(b) $P(A \text{ or } B)$ (b) $\boxed{0.8000}$

$$= P(A) + P(B) - \cancel{P(A \text{ and } B)} \to 0$$

$$= 0.3 + 0.5 = 0.8$$

8. Suppose that $P(A) = 0.4$ and $P(B) = 0.2$. If events A and B are independent, find these probabilities:

(a) $P(A \text{ and } B)$ (a) $\boxed{0.08}$

$= P(A) \cdot P(B) = (0.4) \cdot (0.2) = 0.08$

(b) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (b) $\boxed{0.52}$

$= 0.4 + 0.2 - 0.08$

9. A slot machine has three machine wheels that spin independently. Each has 12 equally likely symbols: 4 bars, 3 lemons and 3 cherries and two bells. If you play, what is the probability that

(a) you get 3 cherries? (a) $\boxed{0.0156}$

P(1st wheel cherry AND 2nd wheel cherry AND 3rd wheel is cherry)

$= \frac{3}{12} \cdot \frac{3}{12} \cdot \frac{3}{12}$

(b) you get no fruit symbols? (b) $\boxed{0.125}$

P(1st wheel bar AND 2nd wheel bar AND 3rd wheel bar)

$= \frac{6}{12} \cdot \frac{6}{12} \cdot \frac{6}{12}$

(c) you get 3 bell symbols? (c) 0.00463

$\frac{2}{12} \cdot \frac{2}{12} \cdot \frac{2}{12}$

10. You bought a new set of four tires from a manufacturer who just announced a recall because 2.5% of those tires are defective. What is the probability that at least one of yours is defective? .025

P(at least one defective) 10. 0.0963

$= P(1 \text{ or more defectives}) = 1 - P(\text{no defectives}) = 1 - P(\text{all 4 tires are good})$

$= 1 - (0.975)^4 = 0.0963$

11. Voter Support for political term limits is strong in many parts of the U.S. A poll conducted by the Field Institute in California showed support for term limits by a 2–1 margin. The results of this poll of $n = 347$ registered voters are given in the table.

	For (F)	Against (A)	No Opinion (N)	Total
Republican (R)	0.28	0.10	0.02	0.40
Democrat (D)	0.31	0.16	0.03	0.50
Other	0.06	0.04	0.00	0.10
Total	0.65	0.30	0.05	1.00

If one individual in drawn at random from this group of 347 people, calculate the following probabilities:

(a) $P(R)$ (a) 0.40

(b) $P(F|R)$ (b) 0.70

(c) $P(D|A)$ (c) 0.5333

(d) $P(F)$ (d) 0.65

(e) $P(R \text{ or } F)$ (e) 0.77

b) $P(F|R) = \dfrac{P(F \text{ and } R)}{P(R)} = \dfrac{0.28}{0.40} = 0.70$

c) $P(D|A) = \dfrac{P(D \text{ and } A)}{P(A)} = \dfrac{0.16}{0.30} = 0.5333$

e) $P(R \text{ or } F) = P(R) + P(F) - P(R \text{ and } F)$

$= 0.40 + 0.65 - 0.28 = 0.77$

Expected Value

$$\boxed{\mu = E(X) = \sum x \cdot P(X)}$$

Standard Deviation

$$\boxed{\sigma = \sqrt{\sum (x-\mu)^2 \cdot P(X)}}$$

12. Find the expected value and standard deviation of the following probability model.

X	3	5	7
$P(X)$	0.25	0.35	0.40

$\boxed{\begin{array}{l} \mu = E(X) = 5.3 \\ \sigma = \sqrt{2.51} \doteq 1.58 \end{array}}$

12.

X	P(x)	$X \cdot P(x)$	$(x-\mu)^2$	$(x-\mu)^2 \cdot P(x)$
3	0.25	0.75	$(3-5.3)^2 = 5.29$	$(5.29)\cdot(0.25) = 1.3225$
5	0.35	1.75	$(5-5.3)^2 = 0.09$	$(0.09)\cdot(0.35) = 0.0315$
7	0.40	2.8	$(7-5.3)^2 = 2.89$	$(2.89)\cdot(0.40) = 1.156$

$\mu = E(x) = \sum x \cdot p(x) = 0.75 + 1.75 + 2.80 = 5.3$

$\sum (x-\mu)^2 \cdot p(x) = 1.3225 + .0315 + 1.156 = 2.51;\ \sigma = \sqrt{2.51}$

13. An insurance policy costs \$100 and will pay policy holders \$10,000 if they suffer a major injury (resulting in hospitalization) or \$3000 if they suffer a minor injury (resulting in lost time for work). The company estimates that each year 1 in every 2000 policy holders have a major injury, and 1 in 500 a minor injury.

(a) Create a probability model for the profit on a policy.

(b) Find the expected value on this policy. (b) \$89

(c) Find the standard deviation. (c) \$260.54

Let X = the discrete random variable representing the company's profit on a single policy.

Outcome	X	P(X)	$X \cdot P(x)$	$(X-\mu)^2 \cdot P(X)$
major injury	$-\$9900$	$\frac{1}{2000}$	$-\$4.95$	$(-9900-89)^2 \cdot \left(\frac{1}{2000}\right) = 49890.0605$
minor injury	$-\$2900$	$1/500 = 4/2000$	$-\$5.80$	$(-2900-89)^2 \cdot \left(\frac{1}{500}\right) = 17868.242$
no injury	$+\$100$	$\frac{1995}{2000}$	$+\$99.75$	$(100-89)^2 \left(\frac{1995}{2000}\right) = 120.6975$

$\mu = E(x) = \sum x \cdot p(x) = -4.95 - 5.80 + 99.75 = \$89$

$\sum (x-\mu)^2 \cdot p(x) = 67{,}879$

$\sigma = \sqrt{67879} \approx \$260.54$

14. Assume that a procedure yields a binomial probability model with a trial repeated $n = 5$ times. Suppose the probability of success on a single trial is $p = 0.47$. Then X counts the number of successes among 5 trials. Describe the distribution (probability model) by filling out the table below. Round calculations to three decimal places.

x	$P(X = x)$
0	0.042
1	0.185
2	0.329
3	0.292
4	0.129
5	0.022

15. College campuses are graying! According to a recent article, one in four college students is age 30 or older. Many of these students are women updating their job skills. Assume that the 25% figure is accurate, that your college is representative of colleges at large, and that you sample $n = 200$ students, recording X, the number of students age 30 or older. What are the mean and standard deviation of X?

$$\boxed{\begin{array}{l} E(X) = \mu = n \cdot p = 50 \\ \sigma = \sqrt{npq} = 6.12 \end{array}}$$

We are told $X =$ the discrete random variable representing the number successes (a person is age 30 or older) in $n = 200$ trials.

Whether or not any student is age 30 or older is independent of whether or not any other student is age 30 or older. Assuming the 25% figure is correct, then asking 200 students if they are age 30 or older is a series of Bernouilli Trials, with success defined as getting a student age 30 or older, and $P(\text{success}) = p = 0.25$. Since $X =$ the number of successes in n trials, the associated probability model is a binomial distribution. Then $E(X) = n \cdot p = (200)(0.25) = 50$ and

$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{(200)(0.25)(0.75)} \simeq 6.12$$

16. Assume that 13% of people are left-handed. If we select 5 people at random, find the probability of each outcome described below.

Geometric —(a) The first lefty is the fifth person chosen. (a) ________

Binomial — (b) There are some lefties in the group of 5 people. (b) ________

Geometric —(c) The first lefty is the second or third person chosen. (c) ________

Binomial —(d) There are exactly 3 lefties in the group. (d) ________

Binomial —(e) There are no more than 3 lefties in the group. (e) ________

These may be considered Bernoulli trials. There are only two possible outcomes, left-handed and not left-handed. Since people are selected at random, the probability of being left-handed is constant at about 13%. The trials are not independent, since the population is finite, but a sample of 5 people is certainly fewer than 10% of all people.

Let X = the number of people checked until the first lefty is discovered.
Let Y = the number of lefties among $n = 5$.

a) Use *Geom*(0.13).
$P(\text{first lefty is the fifth person}) = P(X=5) = (0.87)^4(0.13) \approx 0.0745$

b) Use *Binom*(5,0.13).
$P(\text{some lefties among the 5 people}) = 1 - P(\text{no lefties among the first 5 people})$

$$= 1 - P(Y=0)$$
$$= 1 - \binom{5}{0}(0.13)^0(0.87)^5$$
$$\approx 0.502$$

c) Use *Geom*(0.13).
$P(\text{first lefty is second or third person}) = P(X=2) + P(X=3) = (0.87)(0.13) + (0.87)^2(0.13) \approx 0.211$

d) Use *Binom*(5,0.13).
$P(\text{exactly 3 lefties in the group}) = P(Y=3) = \binom{5}{3}(0.13)^3(0.87)^2 \approx 0.0166$

e) Use *Binom*(5,0.13).
$P(\text{at most 3 lefties in the group}) = P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3)$

$$= \binom{5}{0}(0.13)^0(0.87)^5 + \binom{5}{1}(0.13)^1(0.87)^4$$
$$+ \binom{5}{2}(0.13)^2(0.87)^3 + \binom{5}{3}(0.13)^3(0.87)^2$$
$$\approx 0.9987$$

17. Suppose a certain tennis player makes a successful first serve 70% of the time. Assume that each serve is independent of the others. If she serves 100 times in a match, find the following.

(a) What are the mean and standard deviation of the good first serves expected?

(b) Verify that you can use a Normal model to approximate the distribution of the number of good first serves.

(c) Use the 68–95–99.7 Rule to describe the distribution.

(d) Using the Normal Model, what's the probability she makes <u>at least 65</u> first serves?

(e) Using the Normal Model, what's the probability she makes <u>not more than 75</u> first serves?

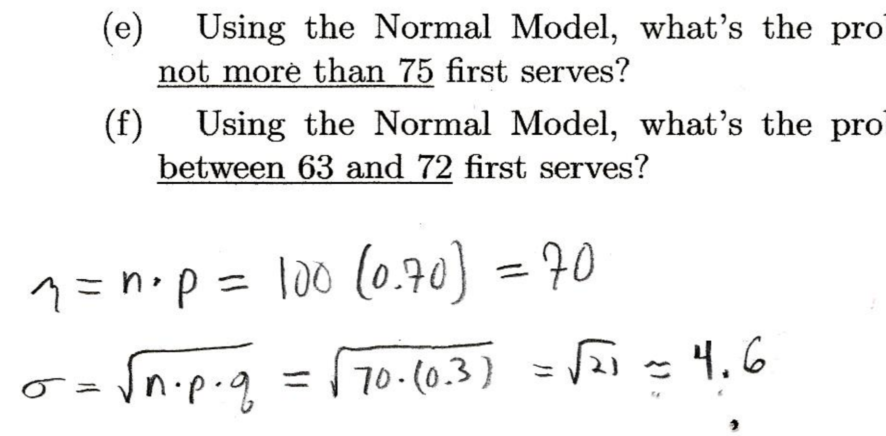
(f) Using the Normal Model, what's the probability she makes <u>between 63 and 72</u> first serves?

a) $\mu = n \cdot p = 100\,(0.70) = 70$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{70 \cdot (0.3)} = \sqrt{21} \simeq 4.6$

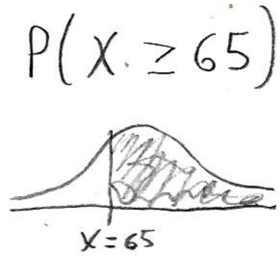
b) $\left[\begin{array}{c} n \cdot p = 70 \geq 10 \\ \text{and} \\ n \cdot q = 30 \geq 10 \end{array}\right] \Longrightarrow$ [The normal model can be used to approximate the binomial model.]

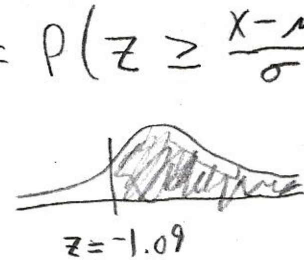
c)



68% of the time, she has between 65 and 74 successful first serves; 95% of the time she has between 60 and 79 successful first serves in 100 trials. 99.7% of the time, she has between 56 and 83 successful first serves

d) $P(X \geq 65) = P\left(z \geq \frac{x-\mu}{\sigma}\right) = P\left(z \geq \frac{65-70}{4.58}\right) \approx P(z \geq -1.09)$

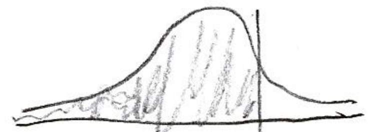




= normalcdf(−1.09, 10)

= 0.8621

$X=75$



$Z=1.09$

e) $P(X \le 75) = P\left(Z \le \frac{x-\mu}{\sigma}\right)$

$= P\left(Z \le \frac{75-70}{4.58}\right) = P(Z \le 1.09)$

$=$ normalcdf$(-10, 1.09) = 0.8621$

f) $P(63 \le X \le 72)$

$= P\left(\frac{x-\mu}{\sigma} \le X \le \frac{x-\mu}{\sigma}\right)$

$= P\left(\frac{63-70}{4.58} \le Z \le \frac{72-70}{4.58}\right)$

$= P(-1.53 \le Z \le 0.44) =$ normalcdf$(-1.53, 0.44)$

$= 0.6070$